How Does Teacher Knowledge in Statistics Impact on Teacher Listening?

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For teaching statistics investigations at primary school level, teacher knowledge has been identified using a framework developed from a classroom based study. Through development of the framework, three types of teacher listening problems were identified, each of which had potential impact on the students' learning. The three types of problems are described, with examples from the classroom along with links to the teacher knowledge framework. It is concluded that teacher knowledge is a necessary condition for avoiding such listening problems.

Teacher listening is known to be a critical factor for successful classroom discourse. The relationship between teacher knowledge and teacher listening, and how the former impacts on the latter, is an important aspect for investigation, and has been the focus of research in mathematics education. Less is known however about the relationship between these two aspects in statistics education. In this paper, teacher listening in relation to teaching statistics through investigations at the primary school level is examined, using data from a classroom based study into teacher knowledge. A number of 'themes' in relation to teacher listening are discussed. Implications of these listening practices are considered in relation to students' learning of statistics.

Literature Review

The importance of establishing effective classroom discourse for enhancing mathematics learning is well recognised. Walshaw and Anthony (2008) indicate that such research in the mathematics education domain is more recent than research about appropriate pedagogical approaches for establishing effective classroom discourse in a generic sense. Teacher practices in relation to listening to students have been found to have an noticeable impact on classroom discourse.

Various aspects of teacher listening have been explored by researchers. For example, Anghileri (2006) identifies types of listening, while Davis (1997) elaborates on levels of listening. However, for teachers to be able to advance students' understanding appropriately, they need to be able deal with students' incorrect responses. This has, according to O'Connor (2001), rarely been the focus of research. Davis's (1997) evaluative listening refers to the teacher being able to compare a student's response with a preconceived answer or standard. With this type of listening, Davis suggests that the teacher is not really interested in what the student is actually saying and is therefore at the lowest level of listening with regard to developing effective classroom discourse. In comparison with evaluative listening, Davis's second level is interpretive listening, in which the teacher makes a more active attempt to connect with and make sense of the student's response, while the highest level which is hermeneutic listening, in which there is a negotiated and participatory interaction between the teacher and student. In the situation of evaluative listening, Davis does not however give consideration to why the teacher does not actively evaluate the student's response. Mercer (1995) describes teacher talk in terms of eliciting comments from students, responding to students, and recapping on students' comments. As part of responding to students, Mercer refers to a range of possible ways of responding such

In J. Dindyal, L. P. Cheng & S. F. Ng (Eds.), Mathematics education: Expanding horizons (Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia). Singapore: MERGA.

as confirming, repeating, paraphrasing or reformulating, elaborating, rejecting, and ignoring. Just as Davis (1997) gives no explanation for why a teacher might not actively evaluate a student's response, Mercer does not suggest any reason why a teacher might ignore a student's response.

Types of 'hearing' have been identified and described by Even and Wallach (Even & Wallach, 2003; Wallach & Even, 2005). The types of hearing are: over-hearing - hearing more than the students are actually saying; under-hearing - missing some of what students say; and compatible hearing - making sense of and connecting with what students say; non-hearing - missing the whole message of the student; and biased hearing - the amount heard depends on who is saying it. Additionally, they consider the reasons behind the various types of hearing. One of numerous factors they give for these different types of hearing is teacher knowledge. In a similar vein, Boaler (2000) argues that the practices within a classroom cannot be separated from the teacher's mathematical content knowledge. O'Connor (2001) suggests that the complexity for the teacher in trying to understand what a student is saying, in keeping track of the sequence of contributions from students, and while thinking about how to respond all within a 'conversationally appropriate' two or three second interval, impacts significantly on the teacher's decision making with regard to a response to the student. So, the type of response that a teacher may give to a student's comment may not be the best or most appropriate because of this decision-making under pressure.

To develop students' mathematical understanding requires teachers to scaffold the students' learning through making connections with students' ideas. This clearly is dependent on teacher listening and teacher knowledge. Such scaffolding includes restructuring (Anghileri, 2006), which involves the teacher making contact with the student's understanding and being able to move it forward. If this does not happen, such as when a teacher does not seek further clarification from a student following his or her contribution to a discussion, it can be referred to as 'pseudo-interaction' or 'by-passing' (Bliss, Askew, & Macrae, 1996). In this situation, the conditions for scaffolding are present but not noticed by the teacher, and consequently, no real interaction occurs between students and teacher. Unfortunately, Bliss and colleagues do not suggest why the teacher does not help the students clarify their thinking. O'Connor (2001), however, posits that a lack of comfort when attempting to make sense might be experienced by the teacher, the student who responded, and the other students, which could contribute to the teacher's failure to seek clarification from the student.

The teaching of statistics is known to present some unique challenges attributable to the nature of statistics and statistical thinking in comparison with mathematics and mathematical thinking. One of the main reasons relates to the deterministic nature of mathematics in comparison with the uncertainty and context-laden nature of statistics (e.g., Cobb & Moore, 1997; delMas, 2004; Moore, 1990; Pereira-Mendoza, 2002). To consider teacher knowledge for statistics requires a different approach from that for other areas of mathematics. One such approach resulted in a framework for describing teacher knowledge through combining statistical thinking (Wild & Pfannkuch, 1999) and components of teacher knowledge, such as described by Ball and colleagues (Ball, Thames, & Phelps, 2005; Hill, Schilling, & Ball, 2004). This framework (Burgess, 2006) provides a way of examining teacher knowledge in statistics, and helps make sense of classroom situations involving teacher listening.

This paper therefore explores the following question: How does the knowledge profile of a teacher relate to the teacher listening to and interpreting students' statements in the classroom?

Method

The research for this paper was conducted in four upper-level primary school classrooms (Years 5-8; either single year level or combined year levels). The data consisted of: videos from a sequence of four lessons in each classroom; and recordings of stimulated recall interviews with each teacher following each lesson, using an edited video of ‘episodes’ from that lesson (between 7 and 29 minutes duration for the edited ‘movie’). Ethics approval and consent from all participants (students, parents/caregivers, and teachers) was gained. The lessons involved statistics investigations based on one unit of work from a website, and used multivariate data (four variables of category data or a mix of category and numeric data).

The analysis of the data involved using the teacher knowledge framework for statistics to identify four components of teacher knowledge from the Ball and colleagues (Ball et al., 2005; Hill et al., 2004) - common knowledge of content, specialised knowledge of content, knowledge of content and teaching, and knowledge of content and students, and the statistical thinking components from Wild and Pfannkuch (1999). A teacher knowledge profile was obtained for each of the four teachers. One such example of a profile is shown in Figure 1:

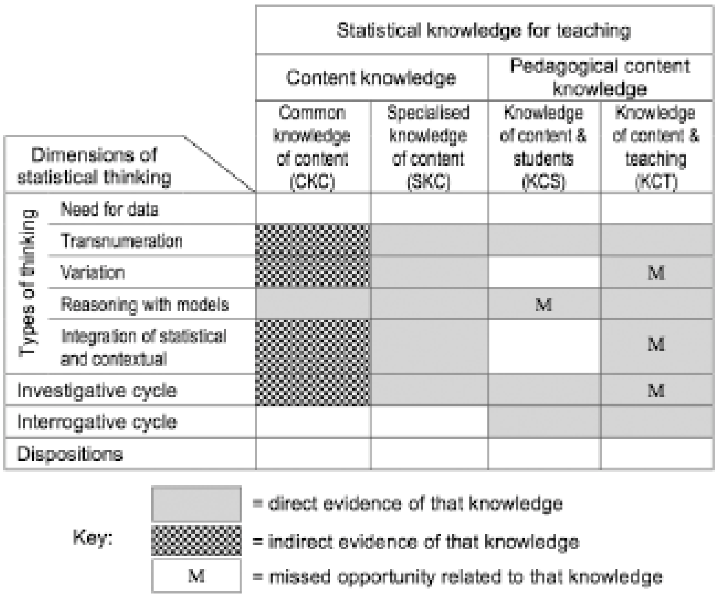

Figure 1. Example of knowledge profile for one teacher

The shaded cells show aspects of knowledge that were evidenced, either directly through teacher comments/explanations/discussion, or indirectly through other related components of knowledge. Additionally, the profile indicates some ‘missed opportunities’, instances in the classroom that revealed the teacher’s lack of relevant knowledge, and which potentially impacted on the students’ opportunity to learn.

Results and Discussion

Through the analysis and profiling of each teacher and in particular the identification of missed opportunities, a theme emerged in relation to teacher listening. Three different types of listening problems were identified. First, after the teacher had listened to a student's statement or question, the response indicated that the teacher had not interpreted the student correctly, and thereby 'answered a different question' from what the student was asking or saying. Second, the teacher did not evaluate an incorrect or 'off-track' response, and did not follow up with any further question nor seek further elaboration from the student that might have helped the student's understanding to develop. Third, if a student gave an incomplete response, the teacher did not ask for more explanation or clarification from the student in order to make sense of what the student was saying. Each of the these three types of listening 'problems' is described, with supporting examples from classroom interactions, and links to relevant literature.

Teacher Responding to Different Question or Statement from the One Asked

In some situations, the teacher did not address the intent of the student's statement or question. One example related to the students considering what data collection question might have resulted in particular data for two different variables. A data card, which showed four data for one student, included the words 'youngest' and 'whistle'. The class were suggesting some possible data collections questions that would have generated these data. The teacher indicated that the correct data question for 'whistle' was, "Can you whistle?" A student questioned this, and the teacher's responded as follows:

> Student 1: How do you know that that question is the right one? Like, all these questions could be different, because, like, … his favourite toy could be whistle …
>
> Teacher: So you think we might need more for "Can you whistle?" So how could we extend that question to make it more specific, then, if you see a problem with that question? What could we add to it? Because we are trying to find out if the person can whistle. Not get a whistle off the teacher or off a toy and whistle. Can you whistle … what do we need to add?
>
> Student 2: Can you whistle using only your mouth?
>
> Teacher: So that means you can't use your fingers as well then? Okay then, "Can you whistle using only your mouth?"

The student's question appeared to be suggesting that there were alternative data questions that could have resulted in the data 'whistle', such as, "What is your favourite toy?" The teacher however missed this point, and seemed to think that the student was suggesting something about where the whistle came from. Clearly, neither the teacher nor the student sought further clarification on each other's comments. The process of considering what data question might have generated particular data is not a typical part of the investigative cycle. Instead, part of the planning phase involves developing data collection questions in order to generate particular data to use within an investigation. However, it became clear that the teacher, by responding differently from what the student had asked, missed an opportunity to engage in a discussion that could have helped students develop their understanding of this necessary part of the investigative cycle. The aspect of teacher knowledge relevant to this is specialised knowledge of content related to the investigative cycle. Specialised knowledge of content relates to the teacher being able to draw on their knowledge of statistics to determine whether a student's answer is reasonable from a statistics point of view, and in this case relates particularly to one aspect of the investigative cycle. Later in the series of lessons, the students were to engage in their own investigation, including planning for data collection, so the teacher missed a learning opportunity in relation to what the students would need at a later stage. Wallach and Even (2005) would classify this teacher listening as ‘under-hearing’. The teacher had heard something about the whistling question, but not heard correctly. It did not result in the teacher misleading students’ understanding of statistics. Even if the teacher had heard correctly and responded appropriately, there is no guarantee that the teacher would have taken the opportunity to develop a discussion/teaching point around the data collection question. As O’Connor (2001) indicated, a teacher’s decision making about responding happens within the pressure of the classroom moment.

The Teacher Does Not Evaluate a Student’s Answer

When a teacher does not evaluate a student’s answer, it could indicate a problem with the teacher’s specialised knowledge of content in relation to various aspects of statistical thinking. One set of classroom exchanges focused on the question, “Are there more whistling right handers or whistling left handers proportionally?”, which was based on a multivariate dataset represented in the following table:

Table 1
Two-way table showing the data related to the question about proportions of whistling right handers and whistling left handers

	Left handed	Right handed	Total
Whistlers	2	15	17
Non-whistlers	1	6	7
Total	3	21	24

After some discussion between some students and the teacher, a student commented:

Student 3: We put 8 out of 21 can whistle for right handed. And 2 out of 3 can [whistle] because there were 3 left handed.

Teacher: There were 3 left handed people? Where did you get your 21 from?

Although the teacher questioned the student about the 21, nothing was raised about the student referring to 8 (which should have been 6) or about the ‘can whistle’ part of the same statement. The student should have said that this number cannot whistle. The conversation continued:

Student 3: um ... out of those …

Student 4: It is supposed to be 24.

Teacher: Okay.

Student 4: Was it 6 or 8 out of right handed who could whistle?

Teacher: Let's see, we'll check with another few groups.

Student 4: There's 8 so it's 3x8 = 24, which is 1/3 of them. And 1 out of 3 left handed ... no 2 out of 3 left handed can whistle.

Teacher: Say that again please.

Student 4: There's 8 right handed people who cannot whistle out of 24 which is 1/3. And there's 1 out of left handed people that cannot whistle which is 1/3.

Teacher: So you've looked at the ones that can't whistle in the results as well, out of the group of right handed people. Is that what you've done?

Student 4 continued with the ‘error’ initiated by Student 3, namely the 6 (or 8) right handed people who could whistle, instead of that number being those who could not whistle. However, later in the exchange, Student 4 had self-corrected, without any intervention from the teacher. Another aspect missed by the teacher was that Student 4 compared the 1/3 **of the class** who were right-handers and could whistle with 1/3 **of the left handers** who could whistle. The comparison was invalid as one was a fraction of the whole class whereas the other was a fraction of a subgroup in the class.

It is clear from the exchanges that the teacher did not refer back to or engage with the data to verify what the students were saying, nor did he ask the students to do the same. Statements involving different parts of two-way classification of data were difficult for the students. As an example, it was not uncommon for students to talk about the proportion of non-whistlers who are boys when they were really referring to the proportion of boys who are non-whistlers (or vice versa). In these situations, the teacher did not necessarily notice and question the students further about it. Creating and referring back to a two-way table representation of the data (such as in Table 1 above) would have assisted the teacher in discussing whether these statements were equivalent. However, such a representation was not used, indicating that specialised knowledge of content in relation to reasoning with models as well as in relation to transnumeration of the data (two cells of the teacher knowledge framework in Figure 1) were missing. It is also an example of what Wallach and Even (2005) refer to as either under-hearing or non-hearing. Wallach and Even suggest that teacher knowledge is a significant factor in the breakdown of teachers really listening to students. To effectively evaluate a student’s response requires specialised knowledge of content.

The Teacher Seeks No Further Clarification from Students

The third type of listening problem identified was when the teacher made no attempt to seek further clarification from a student when it seemed that the answer or explanation was incomplete or quite unclear. One such example was following a question from the teacher as to how it might be possible to investigate whether there was a relationship between the hand-spans and the foot sizes of students. One student suggested that you could “add them together and divide by two”. The teacher did not follow up this suggestion with any questions such as what would be added together or why the average obtained (a meaningless measure in this case) would have helped with investigating the existence of a relationship. Another comment from the student about this was that you could find out “who has the biggest feet … on average”. Again, this statement did not make sense in connection with the investigative question, but was not challenged by the teacher.

In terms of the teacher knowledge profile, these missed opportunities for elaboration indicated a problem with specialised knowledge of content connected with the interrogative cycle and the investigative cycle. The interrogative cycle relates to students making connections between the problem and the data, through seeking possibilities, generating ideas, judging what might work, etc. When thinking about the problem (namely, is there a relationship between the two variables), students need to consider what might be useful for the analysis phase of the investigative cycle and the data might be transnumerated into a useful representation. So when the students made their vague suggestions, the teacher needed to ‘think ahead’ and be able to link their suggestions to the data and the problem, and whether the suggestions would usefully lead somewhere. By not doing so, specialised knowledge of content was not activated in relation to the interrogative and investigative cycles, as well as to transnumeration of data. Therefore three different ‘cells’ of the teacher knowledge framework were implicated as not activated. As with the previous listening problem of not evaluating a student's response, this listening problem could be classified as non- or under-hearing (Wallach & Even, 2005).

Conclusions

The classroom examples described in this paper illustrate three different types of listening problems on the part of the teacher, during a series of statistics investigation lessons. These three problems (responding to something different from what the student has said or asked; not evaluating a student's response; and not seeking clarification about an incomplete explanation or answer) are indicative of different aspects of teacher knowledge for statistics that is missing or not activated, and therefore created a missed opportunity for developing students' understanding of statistics. Although Wallach and Even (2005) suggest that there could be a number of factors that contribute to such listening problems, this study illustrates that teacher knowledge has a significant role in avoiding listening problems, and that various types of knowledge are necessary (but not necessarily sufficient on their own) to avoid such problems.

One teacher in the study did not present any listening problems. For this teacher, when for example, a student's explanation was incomplete, the teacher sought further clarification from the student. This teacher therefore showed sound teacher knowledge in most areas of statistical thinking.

Although the listening problems described and exemplified in this paper did not always impact negatively on the students' learning, it is suggested that the learning was not advantaged in ways that might have been possible. It is important that such listening problems are reduced through teachers developing the appropriate teacher knowledge. As a consequence, the classroom discourse practices and opportunities will be enhanced.

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